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Estimating a Benchmark Term Structure of Interest Rates for Mainland China

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A benchmark term structure of interest rates is fundamental for pricing of bonds and other debt instruments such as bank loans, mortgages, corporate debts and interest rate derivatives. It is also a useful tool for portfolio evaluation, risk management, and monetary policy analysis. This paper tries to estimate a benchmark term structure of interest rates for China using data on treasury bonds and presents an estimated curve with maturities ranging from one to seven years, for which liquidity has built up over the years. A method based on Svensson(1994) is employed to estimate the spot interest rate corresponding to each maturity, dealing particularly with the complication introduced by coupon payments.

For the two selected dates of 8 March 2006 and 25 August 2006, the estimated curves are generally upward-sloping, suggesting that the market expected the short-term interest rates to rise in the period ahead. While the curve had shifted upward over time, reflecting monetary policy tightening by the PBoC, it also became flatter on the latter date. This points to expectations of a smaller increase in future short-term interest rates given the tightening measures already implemented.

1. Introduction

The term structure of interest rates depicts the relationship between spot rates and maturity. A benchmark term structure of interest rates plays a key role in monetary policy transmission and financial intermediation. Many financial institutions, including central banks, monitor the term structure of interest rates closely. There are many potential uses of the term structure of interest rates. First, it provides a benchmark for pricing bonds and other debt instruments such as bank loans, mortgages, and interest rate derivatives. This is particularly important now for China, considering the tremendous efforts the authorities have been taking in developing an active renminbi bond market. Second, the term structure of interest rates can be used for portfolio evaluation and risk management, as it provides a full spectrum of benchmark interest rates that can be used to discount future cash flows and to design hedging strategies. The lack of a reliable term structure of interest rates is

often named as a factor inhibiting the development of renminbi hedging instruments. Third, the term structure of interest rates is a key analytical tool used in analysing monetary and financial issues. For example, it can help policy makers gauge market expectations of future interest rates and inflation rates. The domestic and foreign interest rate differential can also give some indication of market expectation of currency depreciation or appreciation, particularly when the restrictions on renminbi convertibility are relaxed over time.

This paper provides an estimate of the term structure of interest rates for China using data on treasury bonds. Section 2 discusses issues involved in estimating a benchmark term structure curve, particularly problems in using the yield to maturity curve as an approximation. Section 3 briefly introduces the estimation methodology, and discusses the results. Section 4 considers some issues related to the use of the estimated term structure curve. Section 5 concludes.

2. Issues in Estimating a Benchmark Term Structure of Interest Rates

A benchmark term structure of interest rates is often based on treasury bonds or debt instruments issued by the monetary authority as these bonds or debt instruments are normally considered free of credit risk. Both treasury bonds and central bank bills issued by the People's Bank of China (PBoC) are traded in the secondary market. The maturities of treasury bonds range from six months to twenty years, while those for central bank bills are generally less than a year. Other differences include tax treatment of interest payments (interest payments of treasury bonds are tax-exempted, while returns of central bank bills are not) and the pattern of coupon payments (central bank bills have no coupon payments). These differences make it difficult to combine the data to estimate a term structure of interest rates on a full spectrum of maturities. Treasury bonds issued on the Mainland in general have coupon payments except for those with maturities of less than a year. The coupon payments create complications for the estimation, and an essential feature of the estimation method discussed below is to deal with this kind of complications.

In order for the estimated term structure of interest rates to be representative, it is important for the underlying market of treasury bonds to be sufficiently liquid and the prices to be market-based. The market for treasury bonds has been fully liberalised, with prices of the bonds determined by demand and supply. Liquidity has also grown overtime. On average, there are about ten bonds with both bid and ask quotes every trading day.

China Government Securities Depository Trust and Clearing Co. Ltd. (CDC) regularly publishes a yield-to-maturity curve for treasury bonds. It is noted that the yield to maturity is different from the spot interest rate, as the former is calculated by equalising the present value of the principal and coupon payments of a bond to its current price, with the discounting computed using a **constant** yield to maturity. Therefore, the yield to maturity depends on the current price of the bond, the pattern of coupon payments and reinvestments. However, the spot interest rate for a given maturity should not depend on the pattern of coupon payments. If a bond is issued without coupons, the yield to maturity will be the same as the spot interest rate. This is why the term structure of interest rates curve is sometimes called zero-coupon yield curve. Otherwise

the yield to maturity can be significantly different from the spot interest rate for the same maturity.

3. Estimating the Term Structure of Interest Rates

Estimation Method

The method used in estimating the term structure curve in this paper is based on Svensson (1994), which is an extension of Nelson and Siegel (1987). The basic idea of the Svensson method is to assume a single functional form between the spot rate and maturity with unknown parameters. When discounting the coupon payments of a bond, instead of using a constant rate as in the case of calculating the yield to maturity, the Svensson method uses different spot interest rates for different maturities. For example, the first-year coupon payment is discounted using the one year spot interest rate, the second year coupon payment is discounted using the two-year spot interest rate. For any particular bond, the present value of all its coupon payments and principal is calculated using the assumed spot rate function. The difference between this present value and the actual bond price is then calculated. If the market is efficient and the assumed functional form is accurate, this difference should be very small.

In order to estimate the unknown parameters in the functional form, the weighted sum of the squared difference for all bonds with different maturities is minimised. The parameter value that minimises the weighted sum provides an estimate of that parameter. Once all the unknown parameter values are estimated, the spot interest rate can be derived from the functional form. Based on these estimated spot interest rates, the term structure curve can be drawn. This approach has been widely used by central banks (BIS (2005)). Appendix I provides the technical details of the estimation method.

Data

All the bond data are provided by CDC. For this paper, two representative dates, 8 March 2006 and 25 August 2006, are chosen because there are relatively more observations on these two days. Data on closing price, maturity, yield to maturity, coupon rate and coupon frequency are used. In order to make full use of the information available, the estimation includes bonds with different coupon frequencies such as yearly coupon payment, half-yearly coupon payment and no coupon payment.

Results

The data of treasury bonds are concentrated on maturities from one to seven years. There are few observations on bonds with maturities of more than seven years and within one year. To check the robustness of the results, the term structure curve is estimated both with and without data for maturities of longer than seven years. The resulting curves are very close to each other for maturities up to seven years. For maturities of less than one year, the term structure may be better estimated separately using data on central bank bills. In this section, only the term structure curve for maturities from one to seven years is presented based on estimation without using data for maturities beyond seven years. Charts 1 and 2 present the estimated curve for 8 March and 25 August 2006, and Chart 3 compares the two curves. Based on these results, the following observations are drawn.

First, the difference between the estimated spot interest rate and the yield to maturity for the same maturity is

considerable. The largest difference between the two is 12 basis points on 25 August for the maturity of 1.92 years, 22 basis points on 8 March for the maturity of 5.72 years. This supports the case that it is not appropriate to use the yield to maturity to approximate the spot interest rate.

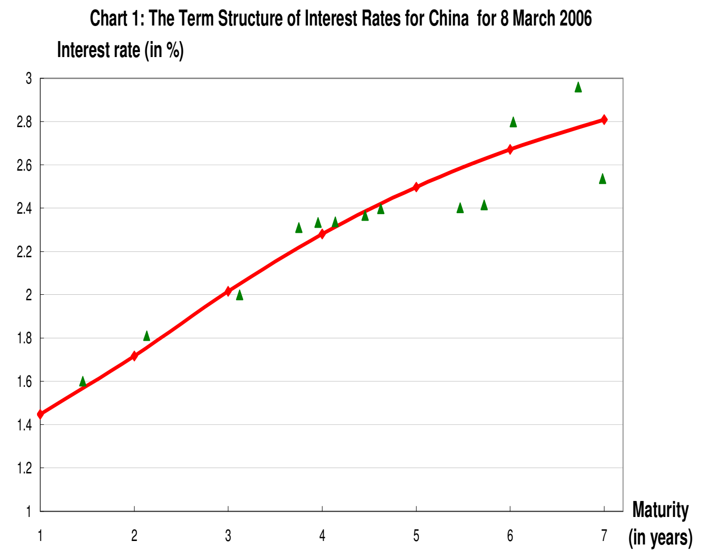
Second, the term structure curve is generally upward-sloping for the two dates estimated. Theoretically, the term structure curve can be upward-sloping, downward-sloping, inverted or flat. The upward-sloping curve suggests that the market expected the future short-term interest rate to go up.

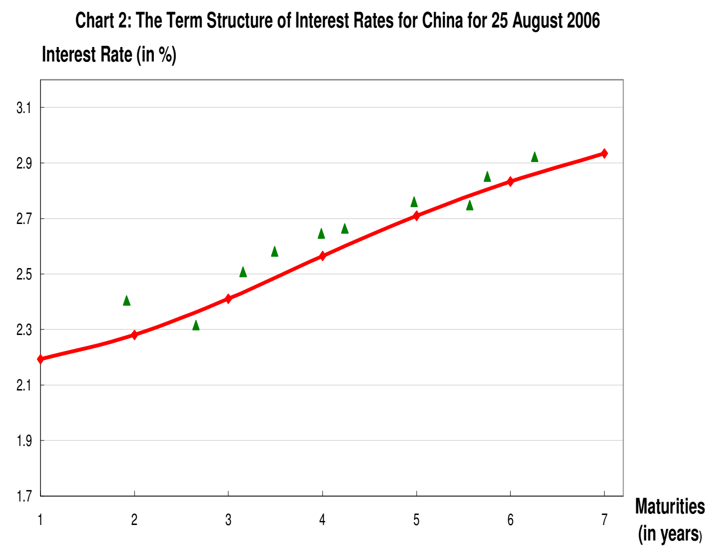
Third, the term structure curve shifted upward from 8 March 2006 to 25 August 2006. The PBoC has intensified monetary tightening since March 2006 through increases in reserve requirements, increases of benchmark lending and deposit rates, issuance of central bank bills, repo transactions and moral suasion. These policy actions raised interest rates at both the short and the long end, resulting in an upward shift of the term structure curve. However, the curve flattened a bit, suggesting that the market expected a smaller rise in future short-term interest rates on 25 August than on 8 March, probably reflecting the effect of the tightening measures implemented during the period.

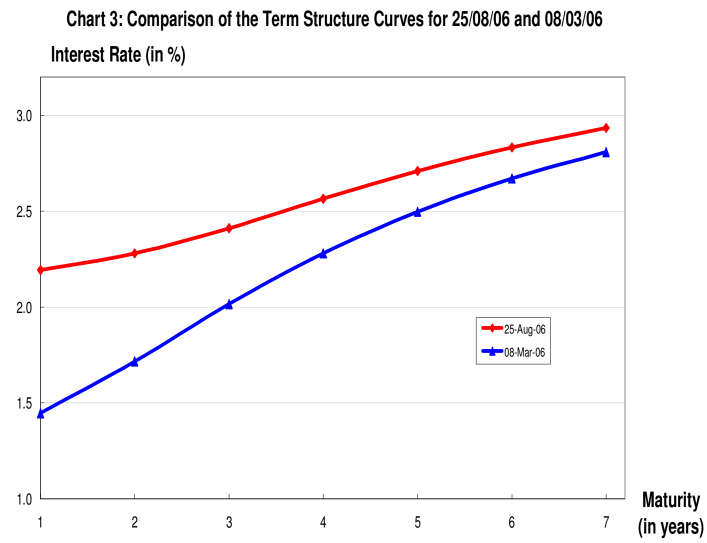
4. Issues in Using the Term Structure of Interest Rates

In using the estimated term structure of interest rates, a number of issues need to be kept in mind.

First is the relevance of the estimated term structure curve as a benchmark for setting other interest rates. China has made significant progress in interest rate







liberalisation in recent years. For example, only a rate ceiling is imposed for bank deposit rate and a rate floor for lending rate. Therefore, the estimated term structure curve, which is a risk-free market-based interest rate structure, could be a useful reference for pricing bank loans (for an overview of China's interest rate structure, see Peng et al. (2006)). On the pricing of corporate bonds, the issue is more complicated. The corporate bond market is still in its early stage of development in China, and interest rates on corporate bonds are still subject to controls. Therefore, the estimated term structure may be less useful as a benchmark, although it should provide a floor to interest rates on corporate bonds.

Second, when using the benchmark term structure for bond pricing, portfolio evaluation and risk management purposes, appropriate adjustment should be made to take into account the liquidity and risk profile of the underlying asset. In the case of China, it is currently difficult to quantify the liquidity and risk profile because of the lack of a reputable local credit rating agency.

Third, while the interest rate differential between renminbi and the US dollar provides information on market expectations of the future change of renminbi-US dollar exchange rate, it should be borne in mind that China still has capital controls, which means the arbitrage mechanism might not work effectively. This is a point to note when the interest rate differential is used to price the forward exchange rate of renminbi.

5. Conclusion

A benchmark term structure of interest rates is fundamental for pricing of bonds and other debt instruments such as bank loans, mortgages, corporate debts and interest rate derivatives. It is also a useful tool for portfolio evaluation, risk management and monetary policy analysis. This paper tries to estimate such a benchmark term structure of interest rates for China using data on treasury bonds. For the two representative dates of 8 March 2006 and 25 August 2006, the resulting curves are generally upward-sloping, suggesting that the treasury bond market expected the short term interest rates to rise in the period ahead. The overall curve shifted upward and flattened during the period, reflecting monetary policy tightening by the PBoC. The difference between the estimated spot rate and the yield to maturity is large for some maturities, pointing to the importance of using a properly estimated interest rate curve to represent the term structure.

References

1. Bank of International Settlements, "Zero-coupon Yield Curves: Technical Documentation", October 2005

2. Nelson, Charles R. and Andrew F. Siegel, "Parsimonious Modelling of Yield Curves", Journal of Business, Vol. 60, No. 4. (October, 1987), Page 473-489.

3. Svensson, Lars E.O., "Estimating and Interpreting Forward Interest rates: Sweden 1992-1994", NBER Working Paper 4871.

4. Peng,Wensheng, Hongyi Chen, Weiwei Fan, "China's Interest Rate Structure and Monetary Policy Implementation", *China Economic Issues*, No.1/2006, June 2006, Hong Kong Monetary Authority.

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About the Series

China Economic Issues provide a concise analysis of current economic and financial issues in China. The series is edited by the External Department.

Appendix I: Estimation Methods for a Term Structure of Interest Rates Curve

The estimation method for a term structure of interest rates curve is generally based on the assumption that there is a functional relationship between the spot rates, or forward rates or discount factors on the one hand and maturities on the other. Discount factors are the present values of treasury bonds with face value of one dollar at different maturities. A discount function $d_{t,m}$ is the collection of discount factors at time t for all maturities m. For establishing a term structure of interest rates curve, the spot rate function, the forward rate function and discount function are equivalent. As long as one of them is known, the other two can be derived from it. The following equations are the relationships between them.

$$d_{t,m} = \exp(-m\, s_{t,m}) \quad \text{or} \quad s_{t,m} = -\frac{1}{m}\ln(d_{t,m}) \tag{A.1}$$

$$s_{t,m} = \frac{1}{m}\int_0^m f(u)\, du \quad \text{or} \quad d_{t,m} = \exp(-\int_0^m f(u)du) \tag{A.2}$$

Where $f(u)$ is the instantaneous forward interest rate, $s_{t,m}$ is the spot rate at time t for maturity m. Both $f(u)$ and $s_{t,m}$ are annualised. Equation (A.1) means that the discount factor over m period is the inverse of the exponential of accumulate interest rate over that period. Equation (A.2) means that the spot rate is the average over the maturity of the accumulation of instantaneous forward rate. The instantaneous forward rate can also be written as the derivative of the spot rate or discount function in the following way.

$$f_{t,m} = s_{t,m} + m\, \dot{s}_{t,m} \quad \text{and} \quad f_{t,m} = -\frac{\dot{d}_{t,m}}{d_{t,m}} \tag{A.3}$$

Where the dots stand for derivatives with respect to time.

Since most bonds are issued with coupons, so the yield to maturity that is observed on the coupon bonds is not a good measure of the time value of money because it depends on the pattern of coupon payments and reinvestments. Therefore it can not be used to discount the future cash flow for bond pricing. It can neither be used to form the term structures of interest rates. The term structure curve has to be estimated using observations from the transaction data of coupon bonds. A variety of techniques have been proposed over the past 35 years. They generally fall into two main approaches: Nelson-Siegel-Svensson approach and spline-based approach. The following provides the explanation of the former approach, which is the approach used in this paper.

It was first proposed in Nelson & Siegel (1987) and was later extended in Svensson (1994). It basically assumes a single functional form for the instantaneous forward interest rate over the entire maturity domain and uses the empirical data to estimate the parameters in the function form. The optimisation can be done through either minimisation of the sum of squared price errors or minimisation of sum of squared yield errors. The following explanation is based on the Svensson (1994).

It is assumed that

$$f_{t,k} = \beta_0 + \beta_1 \exp(-\frac{k}{\tau_1}) + \beta_2 \frac{k}{\tau_1} \exp(-\frac{k}{\tau_1}) + \beta_3 \frac{k}{\tau_2} \exp(-\frac{k}{\tau_2}) \qquad (A.4)$$

Where $f_{t,k}$ denotes the k-period ahead instantaneous forward rate in percentage points. β_0, $\beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ are parameters to be estimated. It should be noted that these parameters are not without constraints. When k approaches to infinity, $f_{t,k}$ should be positive, so β_0 should be greater than zero. τ_1, τ_2 are set to be positive. When k approaches to zero, $f_{t,k} = \beta_0 + \beta_1$ should be positive and equal to overnight spot rate. The data for the overnight interest rate used in the estimation is the overnight repo rate in the inter-bank market in China from database CEIC.

Denote $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ as the parameter set. From equation (A.4), use equation (A.2), we have

$$\begin{aligned} s(m,\beta) = {} & \beta_0 + \beta_1 (1-\exp(-\frac{m}{\tau_1}))(\frac{m}{\tau_1})^{-1} + \beta_2 ((1-\exp(-\frac{m}{\tau_1}))(\frac{m}{\tau_1})^{-1} - \exp(-\frac{m}{\tau_1})) \\ & + \beta_3 ((1-\exp(-\frac{m}{\tau_2}))(\frac{m}{\tau_2}) - \exp(-\frac{m}{\tau_2})) \end{aligned} \qquad (A.5)$$

$$d(m,\beta) = \exp(-m\, s(m,\beta)/100) \qquad (A.6)$$

For any coupon bond j with maturity of m_j years, its theoretical price should be the sum of the present value of coupon payments and final principal. The coupon payment here is assumed to be semi-annual.

$$P_j = c_1\, d(n_j, \beta) + \sum_{k=2}^{g(m_j)} c_k\, d(n_j + 0.5*(k-1)) + V\, d(m_j, \beta) \qquad (A.7)$$

Where $c_1, c_{2,} \cdots, c_k$ are coupon payments. V is the final principal payment. n_j is the time from the settlement to the first coupon payment. $g(m_j)$ is the total number of payment.

$$g(m_j) = \begin{cases} 2m_j \text{ if } 2m_j \text{ is an integer} \\ [2m_j]+1 \text{ if } 2m_j \text{ is not an integer, where } [2m_j] \text{ is the integral part of } 2m_j \end{cases}$$

If the coupon payment is made annually, equation (A.7) should be changed accordingly.

To estimate the parameter vector β, the sum of squared errors of the actual bond prices from the theoretical bond prices are minimised. According to BIS(2005), using bond prices in the estimation irrespective to their durations will lead to over-fitting of the long-term bond prices at the expense of the short-term prices. To

correct this problem, the squared price error is weighted by the inverse of the bond duration. Therefore, the minimisation is done in the following way.

$$\underset{\beta}{Min} \sum_{j=1}^{N} \left\{ (p_j^{obs} - p_j) / \phi_j \right\}^2 \qquad \text{(A.8)}$$

Where p_j^{obs} is the observed bond price, p_j is the theoretical bond price according to equation (A.7), ϕ_j is the duration and N is the total number of bonds.

An alternative way to estimate β is to minimise the sum of squared yield difference. Both the observed yield to maturity and theoretical yield to maturity can be derived from the observed and theoretical prices using the following formula for semi-annual coupon payment bonds.

$$p_j = c_1 \exp(-y_j n_j) + \sum_{k=2}^{g(m_j)} c_2 \exp(-y_j (n_j + 0.5 * (k-1))) + V \exp(-y_j m_j) \qquad \text{(A.9)}$$

Where y_j is the yield to maturity. The minimisation will be

$$\underset{\beta}{Min} \sum_{j=1}^{N} (y_j^{obs} - y_j)^2 \qquad \text{(A.10)}$$

In the original Nelson & Siegel (1987), the last term of the functional form in equation (A.4) does not exist. So the parameter vector β has only four parameters β_0, β_1, β_2, τ_1.

Once all the parameter values are known, the term structure curve can be drawn from equation (A.5).